# Midterm Introduction to Logic (CS and Ma)

Thursday 6 October 2016, 9 – 11 AM

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- ☞ Only write your student number at the top of the exam, not your name, so that we can grade anonymously. Also put your student number at the top of any additional pages.
- ☞ Put the name of your tutorial group ( A, B, C, D, E, F, or G) at the top of the exam and at the top of any additional pages.
- ☞ Leave the first ten lines of the first page blank (this is where the calculation of your grade will be written).
- ☞ Use a blue or black pen (so no pencils, no red pens).
- ☞ With the regular exercises, you can earn 90 points. By writing your student number and tutorial group on all pages, you earn a first 'free' 10 points. With the bonus exercise, you can earn an additional 10 points. The grade is:
  (the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.

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## Good Luck!

**1: Translation into propositional logic (10 points)** Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

a. If she buys fries and she travels by bike, then she also buys ice cream unless it is too hot.

b. Although Máxima did not visit Groningen, she nevertheless visited Warffum or Garmerwolde.

**2: Translation into first-order logic (10 points)** Translate the following sentences to *first-order logic*. Do not forget to provide the translation key (one key for the whole exercise). Represent as much logical structure as possible.

a. David hated neither Elspeth nor Bhaskar, who had both treated him badly.

b. If Bhaskar read the book, then David brought the book from Bhaskar to Anne unless Anne already possessed it.

**3: Formal proofs (30 points)** Give formal proofs of the following inferences. Don't forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.

a.
$$\begin{array}{|l} A \leftrightarrow B \\ C \lor B \\ \hline C \lor A \end{array}$$

b.
$$\begin{array}{|l} C \lor (B \land A) \\ \neg D \to \neg(B \lor C) \\ \hline D \end{array}$$

c.
$$\begin{array}{|l} R(a,b) \land \neg R(b,c) \\ c = b \\ \hline \neg(b = a) \end{array}$$

**4: Truth tables (15 points)** Use *truth tables* to answer the next questions. Make the full truth tables, and do not forget to draw explicit conclusions from the truth tables to motivate your answers.

Order the rows in the truth tables as follows:

| $P$ | $Q$ | $R$ | ... |
|---|---|---|---|
| T | T | T | ... |
| T | T | F | ... |
| T | F | T | ... |
| T | F | F | ... |
| F | T | T | ... |
| F | T | F | ... |
| F | F | T | ... |
| F | F | F | ... |

| Small(c) | Medium(c) | Large(d) | ... |
|---|---|---|---|
| T | T | T | ... |
| T | T | F | ... |
| T | F | T | ... |
| T | F | F | ... |
| F | T | T | ... |
| F | T | F | ... |
| F | F | T | ... |
| F | F | F | ... |

a. Is $((Q \lor R) \land P) \rightarrow Q$ *tautologically equivalent* to $(Q \lor (R \land P)) \rightarrow Q$?

b. Is the sentence (Small(c) $\land$ Medium(c)) $\rightarrow$ $\neg$Large(d) a *logical possibility*? Indicate the spurious rows in the truth table.

**5: Normal forms of propositional logic (15 points)**

a. Provide a disjunctive normal form (DNF) of the sentence:
$(A \lor B) \rightarrow (C \rightarrow D)$.

b. Provide a conjunctive normal form (CNF) of the sentence:
$\neg(A \land B) \leftrightarrow (B \land \neg C)$.

Indicate in both cases the intermediate steps. You do not have to provide justifications for the steps.

**6: Set theory (10 points)** Given are the following three sets: $A = \{\emptyset\}$, $B = \{0, 1, \{1, 0\}\}$ and $C = \{\emptyset, 1, 2, 3\}$. For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

a. $\emptyset \subset A$

b. $A \in C$

c. $(B \cap C) \cup A \subseteq C$

d. $(A \cup B) \cap C = A$

e. $(B \setminus C) \cup A \cup \{3\} \subset C$

f. $\{0, 1\} \not\in (A \cap B) \cup (B \cap C)$

g. $A \setminus C \subseteq A \setminus B$

h. $(C \setminus A) \setminus B = \{2, 3\}$

i. $\emptyset = C \cap B$

j. $A \cap C = \emptyset$

**7: Bonus question (10 points)** Give a formal proof for $(P \land Q) \lor (P \land \neg Q) \lor (\neg P \land Q) \lor (\neg P \land \neg Q)$. Don't forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.